HYENA (HYdro ENergy simulAtor)

Piecewise Linear Approximation of Future Costs

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1. Piecewise Linear Approximation

In [1] the contribution of the reservoirs content in the objective function is the future cost and is represented by:

$$Min\left\{-\sum_{k}\left[\int_{0}^{v_{k,t+1}}\rho_k(x)\cdot\alpha_{s,t+1}(x)\cdot dx\right]\right\} \quad (1)$$

In the linear case $\rho_k(x)$ and $\alpha_{s,t+1}(x)$ are taken out of the integral and (1) simplifies to:

$$Min\left\{-\sum_{k}\rho_k\cdot\alpha_{s,t+1}\cdot v_{k,t+1}\right\} \quad (2)$$

The production coefficients of the k[th] reservoir ρ_k and the water values of the s[th] water area $\alpha_{s,t+1}$ are estimated for every time step in the simulation. $v_{k,t+1}$ is the stored volume in reservoir k at end of stage t.

Using the nonlinear method (1) results in enormous computer time compared to the linear method (2). By using the linear method (2) the solution tends to "swing up and down" between time steps which can be very inconvenient and unrealistic especially for operation of the smaller reservoirs. This has been a long lasting problem for Icelandic hydro power models.

In this report we use a piecewise linear approximation of (1) by introducing the new variables $\Delta v_{k,j,t+1}$:

$$Min\left\{-\sum_{k}0\cdot v_{k,t+1}-\sum_{k}\left[\sum_{j=1}^{Segments}\chi_{k,j}\cdot\Delta v_{k,j,t+1}\right]\right\} \quad (3)$$

With the additional constraints:

$$\sum_{j=1}^{Segments}\Delta v_{k,j,t+1}=v_{k,t+1} \quad (4)$$

$$0\le\Delta v_{k,j,t+1}\le\frac{v_{k,t+1}^{\max}}{Segments} \quad (5)$$

$\chi_{k,j}$ is estimated numerically according to (1).

The variable $v_{k,t+1}$ is kept in the model to be consistent with former results and to simplify programming work.

2. Numerical Results

Figure 1 shows the $\chi_{k,j}$ in one time step of a simulation run for 7 reservoirs and figure 2 shows results of the simulation for 4 water years. In figure 1 Haganga reservoir has the same values as Þorisvatn and is therefore hidden beneath.

Figure 1

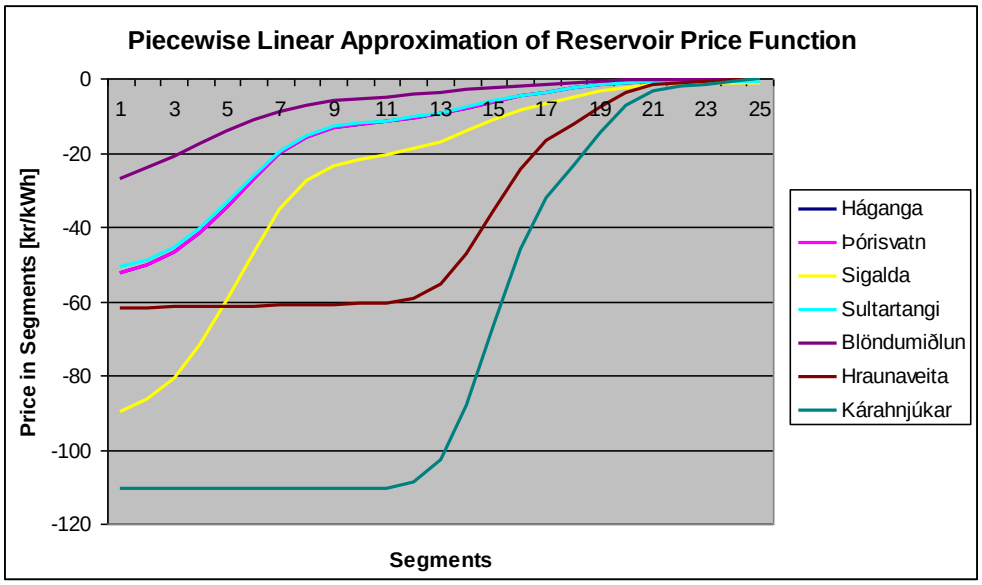
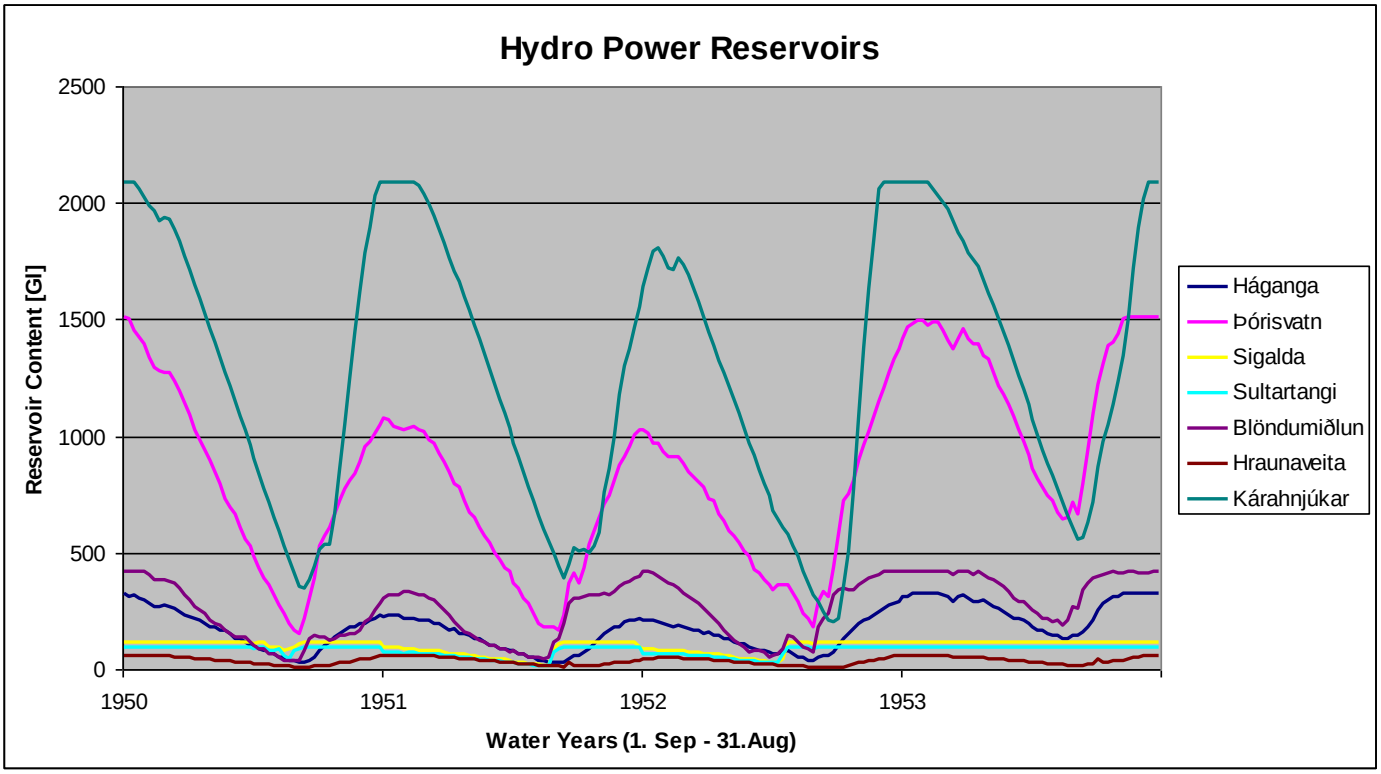


Figure 2



The results in Figure 2 are much more streamlined than former results using (2) with around twice as much computer time, when using 25 segments in the piecewise linear approximation.

The nonlinear case (1) is still almost impossible to use in practice because of excessive computer time.

3. References

[1] HYENA (HYdro ENergy simulAtor). Some Issues in a Simulation Model of a Hydro Thermal Power System. Skuli Johannsson, Annad veldi ehf, Reykjavik, Iceland skuli@veldi.is and Elias B Eliasson The National Power Company, Iceland , elias@lv.is . 01-April-2005.

[2] Elias B Eliasson and Ulfar Linnet, both from Landsvirkjun The National Power Company, contributed to the report.